

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \cdot \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} [N]$$

$$\vec{F} = \frac{q}{4\pi\epsilon_0} \int_V \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} \rho(\vec{r}_1) d^3\vec{r}_1 [N]$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} \rho(\vec{r}_1) d^3\vec{r}_1 \left[ \frac{N}{C} \right]$$

$$\vec{E} = \lim_{q \rightarrow 0} \frac{\vec{F}}{q} \left[ \frac{N}{C} \right]$$

$$\int_V \vec{\nabla} \cdot \vec{E} d^3r = \frac{1}{\epsilon_0} \int \rho d^3r \quad \boxed{\oint_S \vec{E} \cdot \vec{n} \cdot ds = \frac{1}{\epsilon_0} \int \rho d^3r}$$

$$\vec{E} = -\vec{\nabla} \cdot \phi \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = -\vec{\nabla} \cdot \phi \rightarrow \vec{\nabla} \cdot \vec{E} = -\vec{\nabla}^2 \phi \rightarrow \vec{\nabla}^2 \phi = -\frac{\rho}{\epsilon_0}$$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}_1)}{|\vec{r} - \vec{r}_1|} d^3\vec{r}_1 [V]$$

$$W_j = \sum_{i=1}^{j-1} \frac{q_j}{4\pi\epsilon_0} \frac{q_i}{r_{ij}} [J]$$

$$W = \sum_{j=1}^N W_j = \sum_{j=1}^N \sum_{i=1}^{j-1} \frac{q_j}{4\pi\epsilon_0} \frac{q_i}{r_{ij}} = \sum_{j=1}^N \sum_{i=1}^{j-1} q_i \phi_j$$

$$W = \frac{1}{2} \sum_{i \neq j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} = \frac{1}{2} \sum_{i \neq j} q_i \phi_j [J]$$

$$U = \frac{1}{2} \int \rho \phi d^3r$$

$$U = \frac{1}{2} C \Delta \phi^2 = \frac{1}{2} C \frac{Q^2}{C^2} = \frac{1}{2} \frac{Q^2}{C} [J]$$

$$\vec{P} \equiv \lim_{\Delta V \rightarrow 0} \frac{1}{\Delta V} \sum_i \vec{p}_i$$

$$\rho_p = -\vec{\nabla} \cdot \vec{P} \left[ \frac{C}{m^3} \right] \quad \sigma_p = \vec{P} \cdot \vec{n} \left[ \frac{C}{m^2} \right]$$

$$\phi = \frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_p(\vec{r}_1)}{|\vec{r} - \vec{r}_1|} dS_1 + \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}_1)}{|\vec{r} - \vec{r}_1|} d^3\vec{r}_1$$

$$\rho_T = \rho + \rho_p \quad \boxed{\oint_S \vec{D} \cdot \vec{n} \cdot ds = \int \rho d^3r}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \left[ \frac{C}{m^2} \right] \quad \vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$1 + \chi_e = k = \epsilon_r \quad \epsilon_0 \epsilon_r = \epsilon$$

$$C = \frac{Q}{\Delta \phi} = \frac{\sigma S}{\epsilon d} = \frac{S \epsilon}{d} [F] = \left[ \frac{C}{V} \right]$$

$$W_e = \frac{1}{2} \int_V \vec{D} \cdot \vec{E} d^3r = \frac{1}{2} \int \rho \phi d^3r = \frac{1}{2} \int E^2 \epsilon d^3r$$

$$I = \int \vec{J} \cdot \vec{n} ds [A]$$

$$\vec{J} = \sum_i q_i N_i \vec{v}_i \left[ \frac{A}{m^2} \right] \rightarrow \vec{J} = g \vec{E}$$

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Lleis de Kirchhoff:

$$1. \vec{\nabla} \cdot \vec{J} = 0 \rightarrow \int \vec{\nabla} \cdot \vec{J} d^3r = \oint \vec{J} \cdot \vec{n} ds = \sum_i I_i = 0$$

$$2. \int (\vec{\nabla} \times \vec{E}) \cdot \vec{n} ds = \oint_C \vec{E} \cdot d\vec{l} = \sum_i V_i = 0$$

$$\vec{E}_2 = \frac{\mu_0}{4\pi} I_2 I_1 \oint_{C1} \oint_{C2} \frac{d\vec{l}_2 \times [d\vec{l}_1 \times (\vec{r}_2 - \vec{r}_1)]}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \vec{J}(\vec{r}') \times \frac{\vec{r} - \vec{r}_1}{|\vec{r} - \vec{r}_1|^3} d^3r_1 [Tesla]$$

$$\vec{F} = \int \vec{J}(\vec{r}) \times \vec{B}(\vec{r}) d^3r [N]$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) [N]$$

$$\vec{\nabla} \times \vec{B} = -\frac{\mu_0}{4\pi} \vec{\nabla} \frac{\partial}{\partial t} \int_V \frac{\rho(\vec{r}_1)}{|\vec{r} - \vec{r}_1|} d^3r_1 + \mu_0 \vec{J}(\vec{r}) =$$

$$= -\epsilon_0 \mu_0 \frac{\partial}{\partial t} \vec{\nabla} \phi + \mu_0 \vec{J}(\vec{r}) = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}(\vec{r})$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J}(\vec{r}) \cdot \vec{n} ds \rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \cdot \vec{J}$$

$$\rightarrow \vec{\nabla} \cdot \vec{J} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}_1)}{|\vec{r} - \vec{r}_1|} d^3r_1 \rightarrow \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{J} = 0 \rightarrow \vec{\nabla} \times \vec{B} = 0 \rightarrow \vec{B} = -\mu_0 \cdot \vec{\nabla} \phi_m \rightarrow \nabla^2 \phi_m$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{(\vec{m} \times \vec{r})}{r^3}$$

$$\vec{m} \equiv \frac{1}{2} I \oint \vec{r}_1 \times d\vec{l}_1$$

$$\vec{B} = \frac{\mu_0}{4\pi} \left[ \frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right] = -\mu_0 \vec{\nabla} \left( \frac{\vec{m} \cdot \vec{r}}{4\pi r^3} \right)$$

$$\vec{m}_i = \frac{1}{2} \int_{\Delta V} \vec{r}_1 \times \vec{J}(\vec{r}_1) d^3r_1$$

$$\vec{M} \equiv \frac{1}{\Delta V} \sum_i \vec{m}_i = \frac{1}{\Delta V} \frac{1}{2} \int_{\Delta V} \vec{r} \times \vec{J} d^3r [H]$$

$$\vec{J}_M \equiv \vec{\nabla} \times \vec{M} \quad \vec{K}_M \equiv \vec{M} \times \vec{n}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}_m}{|\vec{r} - \vec{r}_1|} d^3r_1 + \frac{\mu_0}{4\pi} \int_S \frac{\vec{K}_m}{|\vec{r} - \vec{r}_1|} d^3r_1$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} \int_V \rho_M(\vec{\mathbf{r}}_1) \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_1}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_1|^3} d^3 r_1 + \frac{\mu_0}{4\pi} \oint_S \sigma_M(\vec{\mathbf{r}}_1) \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_1}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_1|^3} dS_1$$

$$\rho_M = -\vec{\nabla} \cdot \vec{\mathbf{M}} \left[ \frac{A}{m^2} \right] \quad \sigma_M = \vec{\mathbf{M}} \cdot \vec{\mathbf{n}} \left[ \frac{A}{m} \right]$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} \int_V \rho_M \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_1}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_1|^3} d^3 r_1 + \frac{\mu_0}{4\pi} \oint_S \sigma_M(\vec{\mathbf{r}}_1) \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_1}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_1|^3} dS_1 + \mu_0 \vec{\mathbf{M}}$$

$$\vec{\mathbf{J}}_T = \vec{\mathbf{J}}_M + \vec{\mathbf{J}}$$

$$\vec{\mathbf{H}} \equiv \frac{1}{\mu_0} \vec{\mathbf{B}} - \vec{\mathbf{M}} \rightarrow \boxed{\vec{\nabla} \times \vec{\mathbf{H}} = \vec{\mathbf{J}}}$$

$$\int (\vec{\nabla} \times \vec{\mathbf{H}}) \cdot \vec{\mathbf{n}} \cdot d\vec{\mathbf{s}} = \oint \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}}$$

$$\oint \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = \int \vec{\mathbf{J}} \cdot \vec{\mathbf{n}} ds$$

$$\vec{\mathbf{H}} = \frac{1}{4\pi} \int \rho_M(\vec{\mathbf{r}}_1) \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_1}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_1|^3} d^3 r_1 \left[ \frac{A}{m} \right]$$

$$\vec{\nabla} \times \vec{\mathbf{H}} = 0 \quad \vec{\nabla} \cdot \vec{\mathbf{H}} = -\vec{\nabla} \cdot \vec{\mathbf{M}} = \rho_M$$

$$\vec{\mathbf{B}} = \mu_0 (\vec{\mathbf{H}} + \vec{\mathbf{M}}) \quad \vec{\mathbf{M}} = \chi_m \vec{\mathbf{H}}$$

$$1 + \chi_m = \mu_r \quad \mu_0 \mu_r = \mu \quad \vec{\mathbf{B}} = \mu \cdot \vec{\mathbf{H}}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \xi = f.e.m. = -\frac{d}{dt} \int_S \vec{\mathbf{B}} \cdot \vec{\mathbf{n}} ds$$

$$\xi = f.e.m. = -\frac{d\Phi}{dt} \rightarrow \Phi = \vec{\mathbf{B}} \cdot \vec{\mathbf{n}} dS \quad [Wb]$$

$$\boxed{\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}} \quad \oint (\vec{\mathbf{E}} - \vec{\mathbf{v}} \times \vec{\mathbf{B}}) d\vec{\mathbf{l}} = -\int_S \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot \vec{\mathbf{n}} ds$$

$$\vec{\mathbf{M}}_{21} = \frac{\mu_0}{4\pi_1} \oint_{C_2} \oint_{C_1} \frac{d\vec{\mathbf{l}}_2 \cdot d\vec{\mathbf{l}}_1}{|\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1|} = L \quad [H]$$

$$\Phi_{21} = M_{21} \cdot I_1 \quad \Phi_{12} = M_{12} \cdot I_2$$

$$\Phi = LI ; M_{ij} = \frac{d\Phi_{ij}}{dI} \quad [H]$$

$$\boxed{W_m = \frac{1}{2} \int \vec{\mathbf{H}} \cdot \vec{\mathbf{B}} d^3 r}$$

Ecuacions de Maxwell en el buit

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \rho / \epsilon_0 \quad \vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0 \quad \vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

Ecuacions de Maxwell en medis materials

$$\vec{\nabla} \cdot \vec{\mathbf{D}} = \rho \quad \vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0 \quad \nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t}$$

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \quad [N]$$

$$\vec{\mathbf{H}} \equiv \frac{1}{\mu_0} \vec{\mathbf{B}} - \vec{\mathbf{M}} = \frac{\vec{\mathbf{B}}}{\mu} \left[ \frac{A}{m} \right]$$

$$\vec{\mathbf{D}} = \epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}} = \epsilon \vec{\mathbf{E}} \left[ \frac{C}{m^2} \right]$$

$$\phi(\vec{\mathbf{r}}, t) = \frac{1}{4\pi \epsilon_0} \int_V \frac{\rho(\vec{\mathbf{r}}', t')}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} d^3 r'; \quad t' = t - \frac{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|}{c}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}', t')}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} d^3 r'; \quad t' = t - \frac{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|}{c}$$

$$\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \vec{\nabla} \cdot \chi$$

$$\vec{\nabla} \cdot \vec{\mathbf{A}}' = 0 \rightarrow \text{Condicció de Coulomb}$$

$$\nabla^2 \chi = -\vec{\nabla} \cdot \vec{\mathbf{A}}$$

$$\vec{\nabla} \cdot \vec{\mathbf{A}} + \nabla^2 \chi + \epsilon \mu \frac{\partial \phi}{\partial t} = \epsilon \mu \frac{\partial^2 \phi}{\partial t^2} \rightarrow \text{Condicció de Lorentz}$$

Teorema de Poynting

$$\boxed{\int_V \vec{\mathbf{J}} \cdot \vec{\mathbf{E}} d^3 r + \frac{1}{2} \frac{\partial}{\partial t} \int (\vec{\mathbf{E}} \cdot \vec{\mathbf{D}} + \vec{\mathbf{B}} \cdot \vec{\mathbf{H}}) d^3 r = -\oint (\vec{\mathbf{E}} \times \vec{\mathbf{H}}) \cdot \vec{\mathbf{n}} ds}$$

$$\boxed{W = \frac{1}{2} (\vec{\mathbf{E}} \cdot \vec{\mathbf{D}} + \vec{\mathbf{B}} \cdot \vec{\mathbf{H}})}$$

$$\boxed{\vec{\mathbf{S}} = \vec{\mathbf{E}} \times \vec{\mathbf{H}}} \rightarrow \langle S \rangle = \frac{1}{T} \int_0^T S dt$$

$$\nabla^2 \vec{\mathbf{E}} - \epsilon \mu \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} = 0 ; \quad \nabla^2 \vec{\mathbf{H}} - \epsilon \mu \frac{\partial^2 \vec{\mathbf{H}}}{\partial t^2} = 0$$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)} ; \quad \vec{\mathbf{H}} = \vec{\mathbf{H}}_0 e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

$$\boxed{k^2 = \omega^2 \epsilon \mu}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \quad \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega \vec{\mathbf{B}} \quad \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega \vec{\mathbf{D}}$$

$$\boxed{\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{\vec{\mathbf{k}}} \vec{\mathbf{E}}_{\vec{\mathbf{k}}, \omega} e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)} d^2 k d\omega}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{D}}_{\vec{\mathbf{k}}, \omega} = -i \rho_{\vec{\mathbf{k}}, \omega}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{B}}_{\vec{\mathbf{k}}, \omega} = 0$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{E}}_{\vec{\mathbf{k}}, \omega} = \omega \vec{\mathbf{B}}_{\vec{\mathbf{k}}, \omega}$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{H}}_{\vec{\mathbf{k}}, \omega} = -i \vec{\mathbf{J}}_{\vec{\mathbf{k}}, \omega} - \omega \vec{\mathbf{D}}_{\vec{\mathbf{k}}, \omega}$$

Condicions de frontera:

$$1. \quad \vec{\mathbf{n}}_{21} \cdot (\vec{\mathbf{D}}_1 - \vec{\mathbf{D}}_2) = \sigma \rightarrow D_{1n} - D_{2n} = \sigma$$

$$\vec{\mathbf{n}}_{21} \times (\vec{\mathbf{E}}_1 - \vec{\mathbf{E}}_2) = 0 \rightarrow E_{1t} = E_{2t}$$

$$2. \quad \vec{\mathbf{n}}_{21} \cdot (\vec{\mathbf{B}}_1 - \vec{\mathbf{B}}_2) = 0 \rightarrow B_{1n} = B_{2n}$$

$$\vec{\mathbf{n}}_{21} \times (\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2) = \vec{\mathbf{k}} \rightarrow H_{1t} - H_{2t} = \vec{\mathbf{k}}$$

