

$$\vec{p} = m\vec{v}; \quad \vec{L} = \vec{r} \wedge \vec{p}; \quad \vec{N} = \vec{r} \wedge \vec{F}; \quad V(x) = -\int F dx;$$

$$V(x) = \frac{V''(0)}{2} x^2 = \frac{1}{2} kx^2$$

1. OSCIL·LADOR HARMÒNIC SIMPLE

$$F = -kx \begin{cases} m\ddot{x} + kx = 0 \\ m\ddot{x} + \frac{k}{m}x = 0 \end{cases} \rightarrow \begin{cases} x_1 = \cos \omega t \\ x_2 = \sin \omega t \\ \omega = (k/m)^{1/2} \end{cases}$$

$$x = A \cos \omega t + B \sin \omega t \rightarrow x = x_0 \cos \omega t + \frac{V_0}{\omega} \sin \omega t$$

2. OSCIL·LADOR HARMÒNIC ESMORTEÏT

$$F_f = -\alpha \dot{x} \begin{cases} m\ddot{x} + kx + \alpha \dot{x} = 0 \\ F = -kx \\ F = m\ddot{x} \end{cases} \rightarrow \begin{cases} m\ddot{x} + kx + \alpha \dot{x} = 0 \\ \ddot{x} + \frac{\alpha}{m} \dot{x} + \frac{k}{m} x = 0 \end{cases} \left| \begin{array}{l} \gamma = \frac{\alpha}{2m} \\ \omega_0 = (k/m)^{1/2} \end{array} \right.$$

$$x + 2\gamma x + \omega_0^2 x = 0 \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2}$$

a) OSCIL·LADOR HARMÒNIC SUPRAESMORTEÏT ($\gamma > \omega_0$)

$$x(t) = A \exp\left[-(\gamma + \sqrt{\gamma^2 - \omega_0^2})t\right] + B \exp\left[-(\gamma - \sqrt{\gamma^2 - \omega_0^2})t\right]$$

b) OSCIL·LADOR HARMÒNIC INFRAESMORTEÏT ($\gamma < \omega_0$)

$$x(t) = e^{-\gamma t} [A \cos \omega t + B \sin \omega t] \quad \omega^2 = \omega_0^2 - \gamma^2$$

c) OSCIL·LADOR HARMÒNIC CRÍTICAMENT ESMORTEÏT ($\gamma = \omega_0$)

$$x(t) = e^{-\gamma t} [A + Bt]$$

3. OSCIL·LADOR HARMÒNIC FORÇAT

$$m\ddot{x} + \alpha \dot{x} + kx = F(t) \rightarrow \ddot{x} + \frac{\alpha}{m} \dot{x} + \frac{k}{m} x = \frac{F(t)}{m}$$

$$x = e^{-\gamma t} [A \cos \omega t + B \sin \omega t] + \frac{F_1}{m\delta} \cos(\omega_1 t - \theta_1)$$

$$\delta^2 = \omega_0^2 - \omega_1^2 + 4\gamma^2 \omega_1^2$$

ENERGIA OSCIL·LADOR SIMPLE:

$$E = T + V = \frac{1}{2} k a^2 = cte$$

ENERGIA OSCIL·LADOR INFRAESMORTEÏT:

$$E(t + T) = E(t) e^{-2\gamma T}$$

FACTOR QUALITAT: $Q = 2\pi \frac{E}{|\Delta E|} = \frac{\omega_1}{2\gamma}$

POTÈNCIA: $\langle P \rangle = \frac{F_1 |V_{\max}|}{2} \sin \theta_1$; $\sin \theta_1 = \frac{2\gamma \omega_1}{\delta}$; $\cos \theta_1 = \frac{\omega_0^2 - \omega_1^2}{\delta}$

$$\langle P \rangle = m\gamma \omega_1^2 L^2 \quad \text{on } L = \frac{F_1}{m\delta}; \quad \left\langle P \right\rangle_{\text{dissipada per fregament}} = -\alpha V^2 = m\gamma \omega_1^2 L^2$$

ENERGIA OSCIL·LADOR: $E = \frac{1}{2} M \omega_1^2 L^2$

ENERGIA PERDUDA PER PERÍODE:

$$\langle P \rangle \frac{2\pi}{\omega_1} = 2\pi m\gamma \omega_1 L^2 = |\Delta E|$$

RESONÀNCIA: $\omega_0 = \omega_1 \rightarrow \left\{ \begin{array}{l} \delta = 2\gamma \omega_1 \\ \sin \theta_1 = 1 \end{array} \right. \quad A = \frac{F_1}{2m\gamma \omega_1} \xrightarrow{\gamma \rightarrow 0} \infty$

Banda de resonància: $\Delta \omega = 2\gamma$

FORCES CENTRALS

□ Lleis de conservació:

- $E = cte$
- $L = cte$
- Moviment pla

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}; \quad \vec{r} = \vec{r}_1 - \vec{r}_2; \quad \vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \rightarrow \mu \frac{d^2 \vec{r}}{dt^2} = \vec{F}(\vec{r}) \cdot \vec{n}$$

EQUACIÓ DEL MOVIMENT:

$$\begin{cases} m\ddot{r} - m\dot{r}\dot{\theta}^2 = F(r) \\ m\ddot{\theta} + 2m\dot{r}\dot{\theta} = 0 \end{cases} \rightarrow \theta = \theta_0 + \int_0^t \frac{L dt}{mr^2}$$

$$F'(r) = F(r) + \frac{L^2}{mr^3}; \quad V'(r) = V(r) + \frac{L^2}{2mr^2}$$

EQUACIÓ DE L'ÒRBITA: $\frac{d^2 U}{d\theta^2} + U = -\frac{m}{L^2 U^2} F(U)$

VELOCITAT AEROLAR:

$$\frac{\Delta S}{\Delta t} = cte = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \dot{\theta} = \frac{1}{2} \frac{L}{\mu} = cte = \frac{dS}{dt} \quad \left[\frac{S}{T} = \frac{L}{2m} \right] \text{ òrbites tancades}$$

PUNTS DE RETORN: $\frac{1}{r} = -\frac{mk}{L^2} (1 \pm \varepsilon)$;

$\varepsilon =$ excentricitat

1. $k \geq 0, E > 0 \rightarrow \varepsilon > 1 \rightarrow \frac{1}{r} = -\frac{mk}{L^2} (1 - \varepsilon)$ hipèrbola ($-$) $v > v_0$
2. $k < 0, E > 0 \rightarrow \varepsilon > 1 \rightarrow \frac{1}{r} = -\frac{mk}{L^2} (1 + \varepsilon)$ hipèrbola ($+$) $v > v_0$
3. $k < 0, E = 0 \rightarrow \varepsilon = 1 \rightarrow \frac{1}{r} = -\frac{2mk}{L^2} (1)$ paràbola $v = v_0$
4. $k < 0, E < 0 \rightarrow \varepsilon < 1 \rightarrow \frac{1}{r} = -\frac{mk}{L^2} (1 \pm \varepsilon)$ el·lipse $v < v_0$

$\varepsilon = \frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}}$; $2a \equiv r_{\max} + r_{\min}$; $a \equiv$ semieix major; $b \equiv$ semieix menor

HIPÈRBOLA:

$$a(\varepsilon^2 - 1) = r(1 + \varepsilon \cos \theta) \quad \left. \begin{array}{l} + \\ a = -\frac{k}{2E} > 0 \end{array} \right\}; \quad a(\varepsilon^2 - 1) = r(\varepsilon \cos \theta - 1) \quad \left. \begin{array}{l} - \\ a = \frac{k}{2E} > 0 \end{array} \right\}$$

EL·LIPSE:

$$a(1 - \varepsilon^2) = r(1 + \varepsilon \cos \theta); \quad a = \frac{k}{2E} > 0; \quad c = a\varepsilon$$

PARÀBOLA:

$$a = r(1 + \cos \theta); \quad a = \frac{-L^2}{km} > 0$$

$$\frac{a^3}{T^2} = \frac{GM_T}{4\pi^2}; \quad \frac{L}{2m} = \frac{\pi ab}{T}; \quad \tan\left(\frac{\theta}{2}\right) = \left[\frac{mk^2}{2E L^2}\right]^{1/2}; \quad v_{\text{escapament}} = \sqrt{\frac{2GM}{R}}$$

$$dA = 2\pi S dS; \quad \frac{dN}{N} = n dA; \quad \left| \frac{dA}{d\theta} \right| = \frac{2\pi (kq_1 q_2)^2 \sin \theta}{(2m\omega^2)^2 \sin^4(\theta/2)}; \quad \frac{dA}{d\Omega} = \left(\frac{kq_1 q_2}{2m\omega^2} \right) \frac{1}{\sin^4(\theta/2)}$$

$$\vec{R} = \frac{1}{M} \sum_i m_i \vec{r}_i; \quad \vec{R} = \frac{1}{M} \int \vec{r} dm; \quad \vec{p} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M \vec{V}_{CM}$$

$$\vec{L} = \vec{R} \wedge \vec{p} \rightarrow \vec{L} = \vec{R} \wedge \vec{p} + \vec{L}_{CM}$$

$$\frac{d^2 \vec{r}}{dt^2} = \frac{d^2 \vec{r}^2}{dt^2} + 2\vec{\omega} \wedge \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \wedge \vec{r} + \vec{\omega} \wedge (\vec{\omega} \wedge \vec{r}); \quad \vec{a}^* = \vec{g} - 2\vec{\omega} \wedge \vec{v}$$

$$\vec{a}^* = \frac{d^2 \vec{r}}{dt^2} = \vec{g} - 2\vec{\omega} \wedge \vec{v}^* - \vec{\omega} \wedge (\vec{\omega} \wedge \vec{r}); \quad \vec{g}_e = \vec{g} - \vec{\omega} \wedge (\vec{\omega} \wedge \vec{r})$$

$$\begin{cases} M\ddot{R} = \sum F_{\text{ext}} \\ \frac{dL_{\zeta}}{dt} = \sum \vec{N}_0 = \sum \vec{r} \wedge F_{\text{ext}} \end{cases} \quad \left[L_{\zeta} = I_{\zeta} \dot{\theta}; \quad \frac{dL_{\zeta}}{dt} = I_{\zeta} \ddot{\theta} \right]$$

∈ Moments d'inèrcia additius

≠ STEINER: $I = I_G + Md^2$

∠ EIX PERPEENDICULAR: $I_z + I_x + I_y$

$$T_{\text{translació}} = \frac{1}{2} MV_{CM}^2 \quad T_{\text{rotació}} = \frac{1}{2} I \vec{\Omega}^2 \quad T_{\text{TOTAL}} = T_{\text{translació}} + T_{\text{rotació}} = \frac{1}{2} MV_{CM}^2 + \frac{1}{2} I \vec{\Omega}^2$$

TENSOR D'INERCIÀ:

$$I_{ij} = \iiint \rho dV \begin{bmatrix} (x_2^2 + x_3^2) & -x_1 x_2 & -x_1 x_3 \\ -x_1 x_2 & (x_1^2 + x_3^2) & -x_2 x_3 \\ -x_1 x_3 & -x_2 x_3 & (x_2^2 + x_1^2) \end{bmatrix}; \quad \bar{I} \mathbf{a} = \lambda \bar{\mathbf{a}}$$

$$I = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{matrix} (\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \bar{\mathbf{x}}_3): \text{eixos principals d'inèrcia} \\ (I_1, I_2, I_3): \text{moments principals d'inèrcia} \end{matrix}$$

$$I_{ij} = M(\delta_{ij} r^2 - x_i x_j); \quad T = \frac{1}{2} \iiint_V \rho dV \vec{v}^2;$$

$$L = \iiint \rho dV (\vec{r} \wedge \vec{v}) \rightarrow \bar{L} = I \vec{\Omega}; \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\frac{d\bar{L}}{dt} = \bar{N}; \quad \frac{d\bar{L}}{dt} + \vec{\Omega} \wedge \bar{L} = \bar{N}; \quad \begin{matrix} I_3 \dot{\Omega}_3 + (I_2 - I_1) \Omega_1 \Omega_2 = N_3 \\ I_2 \dot{\Omega}_2 + (I_1 - I_3) \Omega_1 \Omega_3 = N_2 \\ I_1 \dot{\Omega}_1 + (I_3 - I_2) \Omega_1 \Omega_3 = N_1 \end{matrix}$$

$$E_{\text{CINÈTICA}} + E_{\text{POTENCIAL}} = \frac{k}{2} A^2 = cte; \quad \det[a - \omega^2 \mathbf{I}]$$

$$\frac{\partial^2 U}{\partial t^2} = \frac{T}{\rho} \frac{\partial^2 U}{\partial x^2} \quad \left| \begin{matrix} T \equiv \text{força; tensió} \\ \rho \equiv \text{densitat lineal massa} \\ v \equiv \text{velocitat} \end{matrix} \right. \quad \frac{\partial^2 U}{\partial t^2} = v^2 \frac{\partial^2 U}{\partial x^2}$$

$$U(x, t) = \left[C \cos \frac{\omega x}{v} + D \sin \frac{\omega x}{v} \right] [A \cos \omega t + B \sin \omega t]$$

$$U(x, t) = \sin \frac{\omega x}{v} [A \cos \omega t + B \sin \omega t]$$

$$\left| \begin{matrix} A \sin \theta + B \cos \theta = \sin(\theta + \delta) \\ C^2 = A^2 + B^2, \cos \delta = A/C, \sin \delta = B/C \end{matrix} \right.$$

$$\lambda = \frac{2\pi v}{\omega} = \frac{2l}{n}; \quad E = \frac{m}{2} \omega^2 A^2 \rightarrow \rho dx: \quad dE = \frac{1}{2} \rho dx \omega^2 A^2$$

$$\# \text{ones: } v = \frac{1}{\lambda}; \quad P = \frac{dE}{dt} = \frac{v dE}{dx} = \frac{1}{2} \rho v \omega^2 A^2; \quad k = \frac{\omega}{v} = \frac{2\pi}{\lambda};$$

$$\nabla^2 U = \frac{1}{v^2} \frac{\partial^2 U}{\partial t^2}; \quad f = \frac{\omega}{2\pi} = \frac{v}{\lambda} = T^{-1}; \quad T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{\lambda}{v}$$

$$U(\vec{r}, t) = f(\vec{U} \cdot \vec{r} + vt) + g(\vec{n} \cdot \vec{r} - \theta t)$$

$$U(\vec{r}, t) = A \sin(\vec{k} \cdot \vec{r} - \omega t + \delta)$$

$$\vec{k} = k \vec{n} \quad \nabla^2 U = -k^2 U$$

$$\frac{\partial^2 U}{\partial t^2} = -\omega^2 U$$