

1. FUNCIONS EXPONENCIALS

$$e^z = e^{x+iy} = e^x (\cos y + i \sin y)$$

$$z^c = e^{c \cdot \ln z} = \exp[c \cdot \ln z]; \quad z, c \in \mathbb{C}$$

2. FUNCIONS TRIGONOMETRIQUES

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sec z = \frac{1}{\cos z} = \frac{2}{e^{iz} + e^{-iz}}$$

$$\csc z = \frac{1}{\sin z} = \frac{2i}{e^{iz} - e^{-iz}}$$

$$\tan z = \frac{\sin z}{\cos z} = \frac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}$$

$$\cot z = \frac{1}{\tan z} = \frac{\cos z}{\sin z} = \frac{i(e^{iz} + e^{-iz})}{e^{iz} - e^{-iz}}$$

I) $\sin^2 z + \cos^2 z = 1$

II) $1 + \tan^2 z = \sec^2 z$

III) $1 + \cot^2 z = \csc^2 z$

IV) $\sin(-z) = -\sin z$

V) $\cos(-z) = \cos z$

VI) $\tan(-z) = -\tan z$

VII) $\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \cos z_1 \sin z_2$

VIII) $\cos(z_1 \pm z_2) = \cos z_1 \cos z_2 \mp \sin z_1 \sin z_2$

IX) $\tan(z_1 \pm z_2) = \frac{\tan z_1 \pm \tan z_2}{1 \mp \tan z_1 \tan z_2}$

3. FUNCIONS HIPERBÒLIQUES

$$\sinh z = \frac{e^z - e^{-z}}{2}$$

$$\cosh z = \frac{e^z + e^{-z}}{2}$$

$$\operatorname{sech} z = \frac{1}{\cosh z} = \frac{2}{e^z + e^{-z}}$$

$$\operatorname{csch} z = \frac{1}{\sinh z} = \frac{2}{e^z - e^{-z}}$$

$$\tanh z = \frac{\sinh z}{\cosh z} = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$\operatorname{coth} z = \frac{1}{\tanh z} = \frac{\cosh z}{\sinh z} = \frac{e^z + e^{-z}}{e^z - e^{-z}}$$

I) $\sinh^2 z - \cosh^2 z = -1$

II) $1 - \tanh^2 z = \operatorname{sech}^2 z$

III) $\operatorname{coth}^2 z - 1 = \operatorname{csch}^2 z$

IV) $\sinh(-z) = -\sinh z$

V) $\cosh(-z) = \cosh z$

VI) $\tanh(-z) = -\tanh z$

VII) $\sinh(z_1 \pm z_2) = \sinh z_1 \cosh z_2 \pm \cosh z_1 \sinh z_2$

VIII) $\cosh(z_1 \pm z_2) = \cosh z_1 \cosh z_2 \pm \sinh z_1 \sinh z_2$

$$\text{ix) } \tanh(z_1 \pm z_2) = \frac{\tanh z_1 \pm \tanh z_2}{1 \pm \tanh z_1 \tanh z_2}$$

$$\begin{aligned} \sin iz &= i \sinh z \\ \sinh iz &= i \sin z \end{aligned}$$

$$\begin{aligned} \cos iz &= \cosh z \\ \cosh iz &= \cos z \end{aligned}$$

$$\begin{aligned} \tan iz &= i \tanh z \\ \tanh iz &= i \tan z \end{aligned}$$

4. FUNCIONS LOGARÍTIQUES

$$\ln z = \ln r + i(\theta + 2\pi n) = \ln|z| + i \arg z = \ln r + i\theta^1$$

$$\log_a^z = \frac{\ln z}{\ln a}$$

5. FUNCIONS TRIGONOMÈTRIQUES INVERSES

$$\sin^{-1} z = 2\pi k + \frac{1}{i} \ln\left(iz + \sqrt{1-z^2}\right)$$

$$\csc^{-1} z = 2\pi k + \frac{1}{i} \ln\left(\frac{i + \sqrt{z^2-1}}{z}\right)$$

$$\cos^{-1} z = 2\pi k + \frac{1}{i} \ln\left(z + \sqrt{z^2-1}\right)$$

$$\sec^{-1} z = 2\pi k + \frac{1}{i} \ln\left(\frac{1 + \sqrt{z^2-1}}{z}\right)$$

$$\tan^{-1} z = \pi k + \frac{1}{2i} \ln\left(\frac{1+z}{1-z}\right)$$

$$\cot^{-1} z = \pi k + \frac{1}{2i} \ln\left(\frac{z+i}{z-i}\right)$$

6. FUNCIONS HIPERBÒLIQUES INVERSES

$$\sinh^{-1} z = 2\pi ki + \ln\left(z + \sqrt{z^2+1}\right)$$

$$\operatorname{csch}^{-1} z = 2\pi ki + \ln\left(\frac{1 + \sqrt{z^2+1}}{z}\right)$$

$$\cosh^{-1} z = 2\pi ki + \ln\left(z + \sqrt{z^2-1}\right)$$

$$\operatorname{sech}^{-1} z = 2\pi ki + \ln\left(\frac{1 + \sqrt{1-z^2}}{z}\right)$$

$$\tanh^{-1} z = \pi ki + \frac{1}{2} \ln\left(\frac{1+z}{1-z}\right)$$

$$\operatorname{coth}^{-1} z = \pi ki + \frac{1}{2} \ln\left(\frac{z+1}{z-1}\right)$$

¹ $r = \sqrt{a^2 + b^2}$ i $\theta = \arctan \frac{b}{a}$

